Supplement to "Employment, Hours of Work and the Optimal Taxation of Low Income Families"

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These online appendices (i) present some additional parameter and results tables, (ii) describe the robustness of our optimal tax simulations with a finer discretisation of work hours, (iii) describe how random measurement error affects the size and placement of the optimal hours bonus, and (iv) quantify the extent that the requirement that no individual is made worse off would act as a constraint on the social welfare maximisation problem.

C Additional Parameter and Results Tables

In Table S1 we present the main parameters of Family Credit and Working Families' Tax Credit (WFTC) for the period April 1999–June 2002; Table S2 presents the maximum likelihood estimates from our structural labour supply model; Table S3 shows the fit to empirical child care usage (panel S3a) and hours (panel S3b) as a function of maternal hours of work; Table S4 shows the simulated impact of WFTC.

D Discrete Hours Sensitivity Analysis

Here we explore the sensitivity of our main simulation results with respect to the number of hours points available. The results reported here double the number of positive hours points (so a total of 11 discrete hours points) and re-estimate the structural model using these.[1] With the new set of parameter estimates, we again simulate a set of optimal tax schedules. The pure earnings schedules are very similar to those obtained with 6 discrete hours points; there are very similar levels of out-of-work income, and marginal rates from moderate earnings levels. The only notable difference is that the marginal tax rates in the first bracket are now slightly higher, while those in the second bracket are slightly lower.

The same general findings are true in the simulations with hours of work bonuses (both fixed, and with optimal hours bonus placement). Moreover, both the size and placement of these hours contingent payments are essentially the same as before. Full results from this exercise are presented in Table S5.

E Random Measurement Error

In section 7.5 from the main text we presented results where individuals were able to directly misreport their hours of work to the tax authorities at some utility cost. We now consider an

[1] The discrete points are now placed at 0, 5, 10, 14.5, 19, 22.5, 26, 29.5, 33, 36.5, and 40.

Table S1: Parameters of FC/WFTC

	April 1999 (FC)	October 1999 (WFTC)	June 2000 (WFTC)	June 2002 (WFTC)
Basic Credit	49.80	52.30	53.15	62.50
Child Credit				
under 11	15.15	19.85	25.60	26.45
11 to 16	20.90	20.90	25.60	26.45
over 16	25.95	25.95	26.35	27.20
30 hour credit	11.05	11.05	11.25	11.65
Threshold	80.65	90.00	91.45	94.50
Taper rate	70% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance
Childcare	Expenses up to £60 (£100) for 1 (more than 1) child under 12 disregarded when calculating income	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £135 (£200) for 1 (more than 1) child under 15

Notes: All monetary amounts are in pounds per week and expressed in nominal terms. Minimum FC/WFTC award is 50p per week in all years above.

alternative scenario where the tax authorities observe some purely noisy measure of actual hours h. There is no direct utility cost associated with this, and reported hours of work h_R are not chosen by the individual. Rather, we allow for random measurement error by adding an independent and normally distributed error term ν to work hours h to form a pseudo reported hours measure, $\tilde{h}_R = h + \nu$. Actual reported hours h_R are then given by the nearest discrete hours point in the set of hours $\mathcal{H}_{++}$. We assume that ν has zero mean, and in Table S6 we show how the size of the hours bonus and the associated welfare gain, vary as the standard deviation of the measurement error term σ_ν increases in value. A clear pattern emerges. Across all values of θ, the size of the optimal hours bonus declines as reported hours become less informative. Furthermore, the placement of the optimal hours rule is reduced by a single discrete hours category for relatively high values of σ_ν (although a non-monotonic relationship is obtained in the case that $\theta = -0.2$). In the simulations where the standard deviation of the error term is between 4 and 8 (so that a single standard deviation results in reported hours differing from actual hours by a single category), the welfare gain from using hours information falls by between around 20% and 40%. The presence of random measurement error clearly reduces the desirability of conditioning upon hours, and if it is modest or large in size, then the welfare gains that are achievable are much reduced.

F The Constraints on Social Welfare Maximisation

The requirement that no individual is made worse off following a tax reform is a demanding criterion, particularly in the presence of preference heterogeneity. In this appendix we seek to quantify the extent to which imposing this requirement may restrict the potential for social welfare improving reforms. To do this, we first perform a similar exercise to that in section 7 from the main text by calculating the social welfare maximizing tax schedule (under the

Table S2: Maximum likelihood estimation results

Preference parameters

	constant	youngest child 0–4	youngest child 5–10	number of children -1	age	compulsory schooling	non-white	London	WFTC period	year 2000
α_y	1.570	-0.441	-0.171	0.018	-0.021	-0.091	–	–	–	–
	(0.128)	(0.119)	(0.096)	(0.039)	(0.007)	(0.094)				
α_l	2.673	0.251	0.203	0.132	-0.035	-0.341	–	–	–	–
	(0.117)	(0.125)	(0.113)	(0.033)	(0.006)	(0.070)				
θ_y	0.301	–	–	–	–	–	–	–	–	–
	(0.085)									
θ_l	1.000	–	–	–	–	–	–	–	–	–
	(–)									
α_f	0.295	0.164	0.029	0.057	0.005	0.072	-0.078	0.261	–	–
	(0.076)	(0.089)	(0.068)	(0.033)	(0.005)	(0.063)	(0.049)	(0.044)		
η	0.982	–	–	–	0.017	-0.116	0.544	–	-0.438	0.388
	(0.208)				(0.009)	(0.161)	(0.181)		(0.117)	(0.134)
σ_y	0.668	–	–	–	–	–	–	–	–	–
	(0.050)									
σ_η	2.182	–	–	–	–	–	–	–	–	–
	(0.195)									
ρ_{yw}	0.241	–	–	–	–	–	–	–	–	–
	(0.042)									

Continued . . .

Table S2: (continued)

Childcare parameters

	1 child youngest age 0–4	1 child youngest age 5–10	1 child youngest age 11–1	2+ children youngest age 0–4	2+ children youngest age 5–10	2+ children youngest age 11–1
γ_c	5.697	-6.371	-26.633	7.237	-22.996	-57.585
	(1.917)	(1.371)	(4.966)	(3.435)	(3.041)	(10.100)
β_c	0.694	0.654	0.283	1.180	1.270	0.640
	(0.064)	(0.047)	(0.150)	(0.131)	(0.109)	(0.301)
σ_c	13.234	11.779	24.528	27.206	27.428	42.603
	(0.474)	(0.314)	(2.246)	(0.941)	(0.872)	(3.751)
$\Pr(p_{cc}^1)$	0.179	0.173	0.145	0.152	0.133	0.175
	(0.019)	(0.018)	(0.036)	(0.019)	(0.016)	(0.048)
$\Pr(p_{cc}^2)$	0.206	0.181	–	0.192	0.147	–
	(0.021)	(0.019)	–	(0.023)	(0.018)	–
$\Pr(p_{cc}^3)$	0.244	0.191	–	0.289	0.162	–
	(0.024)	(0.020)	–	(0.030)	(0.020)	–
p_{cc}^1	0.000	0.000	0.000	0.000	0.000	0.000
p_{cc}^2	0.937	0.804	1.887	0.516	0.570	1.658
p_{cc}^3	2.172	1.594	–	1.547	1.474	–
p_{cc}^4	3.440	2.579	–	2.949	2.474	–

Wage equation

constant	education	age	age squared	non-white	London	1998	1999	2000	2001	2002	σ_w
-0.010	0.097	0.050	-0.051	-0.046	0.192	-0.005	0.025	0.129	0.146	0.144	0.404
(0.067)	(0.004)	(0.012)	(0.017)	(0.026)	(0.024)	(0.025)	(0.029)	(0.025)	(0.023)	(0.023)	(0.005)

Notes: All parameters estimated simultaneously by maximum likelihood, using FRS data and with sample selection as detailed in section 5.1 from the main text. Standard errors calculated using the outer product of gradients method. Incomes are expressed in hundreds of pounds per week in April 2002 prices. Age and age squared are defined in terms of deviations from the median value; age squared is divided by one hundred. Compulsory schooling is equal to 1 if the individual completed school at age 16 or above. Education measures age that education was completed. London is equal to one if resident in the Greater London area. WFTC period is equal to one if individual is interviewed post-October 1999. Standard errors are presented in parentheses.

Table S3: Predicted and empirical childcare use and hours by age of youngest child

(a) Childcare use

	All		0–4		5–10		11–18	
	Predicted	Empirical	Predicted	Empirical	Predicted	Empirical	Predicted	Empirical
10 hours	0.464	0.385	0.771	0.685	0.379	0.316	0.139	0.119
	(0.015)	(0.022)	(0.023)	(0.038)	(0.022)	(0.031)	(0.024)	(0.031)
19 hours	0.594	0.618	0.879	0.892	0.564	0.599	0.167	0.207
	(0.010)	(0.016)	(0.011)	(0.017)	(0.015)	(0.024)	(0.018)	(0.029)
26 hours	0.630	0.554	0.933	0.887	0.699	0.622	0.191	0.118
	(0.008)	(0.025)	(0.006)	(0.031)	(0.011)	(0.036)	(0.014)	(0.031)
33 hours	0.651	0.618	0.967	0.913	0.811	0.782	0.218	0.202
	(0.009)	(0.021)	(0.004)	(0.025)	(0.011)	(0.027)	(0.014)	(0.030)
40 hours	0.655	0.623	0.985	0.886	0.893	0.850	0.246	0.242
	(0.013)	(0.017)	(0.003)	(0.024)	(0.010)	(0.020)	(0.021)	(0.024)

(b) Childcare hours

	All		0–4		5–10		11–18	
	Predicted	Empirical	Predicted	Empirical	Predicted	Empirical	Predicted	Empirical
10 hours	18.656	13.543	23.730	14.738	13.565	12.398	15.861	10.423
	(0.736)	(0.828)	(1.021)	(1.065)	(0.623)	(1.422)	(1.534)	(2.808)
19 hours	22.010	21.538	27.929	27.607	16.466	15.945	16.655	13.268
	(0.495)	(0.664)	(0.757)	(0.972)	(0.482)	(0.848)	(1.347)	(1.422)
26 hours	23.423	24.050	32.018	28.596	19.310	21.098	17.195	17.288
	(0.399)	(1.200)	(0.715)	(1.674)	(0.425)	(1.644)	(1.250)	(6.391)
33 hours	24.821	27.740	35.871	37.776	22.526	22.728	17.510	21.061
	(0.484)	(0.947)	(0.850)	(1.598)	(0.494)	(1.067)	(1.218)	(2.821)
40 hours	26.406	32.406	39.964	45.781	26.298	28.234	17.807	20.332
	(0.689)	(1.046)	(1.155)	(2.274)	(0.706)	(1.108)	(1.271)	(1.872)

Notes: Empirical frequencies calculated using FRS data with sample selection as detailed in Section 5.1. The discrete points 0, 10, 19, 26, 33 and 40 correspond to the hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively. Predicted frequencies are calculated using FRS data and the maximum likelihood estimates from Table S2. Standard errors are in parentheses, and calculated for the predicted frequencies by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table S4: Impact of reforms: 1997-2002

	1997 system	2002 system	change
0 hours	0.546	0.493	-0.053
	(0.007)	(0.007)	(0.007)
10 hours	0.079	0.079	0.000
	(0.003)	(0.003)	(0.001)
19 hours	0.105	0.116	0.010
	(0.003)	(0.003)	(0.004)
26 hours	0.076	0.090	0.014
	(0.002)	(0.002)	(0.002)
33 hours	0.082	0.104	0.022
	(0.002)	(0.002)	(0.002)
40 hours	0.112	0.119	0.007
	(0.004)	(0.004)	(0.001)
Take-up rate	0.697	0.808	0.111
	(0.013)	(0.014)	(0.017)

Notes: impact of tax and transfer system reforms on hours of work and take-up simulated using FRS 2002 data by replacing actual 2002 tax systems with the April 1997 tax system. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

same set of redistributional parameters that was previously considered) subject to the usual incentive compatibility constraints and government revenue constraint. We then proceed to calculate the increase in revenue that is required such that this same level of social welfare is achieved, but subject to the additional requirement that no individual is made worse off relative to the actual tax and transfer.[2]

For each value of the redistributional taste parameter θ we conduct four sets of simulations; when individuals are made no worse off conditional on the full set of observable and unobservable characteristics $(X, \epsilon, \varepsilon)$ both with and without possible hours rules, and also when we only condition on demographics and productive capacity (X, ϵ_w) (again, with and without possible hours rules). The results of this exercise are presented in Table S7, which shows the proportional increase in required government expenditure. The table shows that the constraint that no individual is made worse off would impose a significant restriction on the social welfare maximisation problem. Moreover, the required increase in expenditure is higher when the government values redistribution less.[3]

[2]The "unconstrained" maximisation problem that we consider here differs slightly from that considered in section 7 from the main text. In particular, the tax schedule is now constructed in the same way as when we were examining Pareto improving tax reforms.

[3]Relative to the actual tax and transfer system, the welfare maximisation problem without utility constraints reduces out-of-work income by a larger amount when the government has less redistributional concern. Since such reductions are not possible once we impose these utility constraints, the required increase in expenditure is higher when we consider higher values of the redistributional taste parameter θ.

Table S5: Optimal tax schedules with 11 discrete points

Weekly	No hours			19 hours			Optimal hours		
Earnings	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$
0–50	0.263	0.321	0.273	0.407	0.458	0.337	0.235	0.287	0.263
	(0.039)	(0.052)	(0.061)	(0.048)	(0.060)	(0.068)	(0.042)	(0.054)	(0.063)
50–100	0.205	0.069	-0.063	0.775	0.605	0.175	0.392	0.274	0.010
	(0.032)	(0.037)	(0.050)	(0.024)	(0.026)	(0.060)	(0.028)	(0.030)	(0.043)
100–150	0.300	0.193	-0.067	0.444	0.314	-0.042	0.533	0.443	0.129
	(0.017)	(0.021)	(0.039)	(0.014)	(0.018)	(0.041)	(0.012)	(0.015)	(0.033)
150–200	0.420	0.335	0.007	0.460	0.372	0.023	0.636	0.558	0.174
	(0.019)	(0.021)	(0.035)	(0.018)	(0.021)	(0.037)	(0.018)	(0.022)	(0.043)
200–250	0.517	0.451	0.085	0.542	0.474	0.092	0.609	0.557	0.216
	(0.017)	(0.018)	(0.042)	(0.017)	(0.018)	(0.043)	(0.020)	(0.022)	(0.051)
250–300	0.508	0.456	0.112	0.524	0.470	0.119	0.708	0.660	0.297
	(0.023)	(0.023)	(0.038)	(0.024)	(0.025)	(0.039)	(0.029)	(0.031)	(0.057)
300–350	0.552	0.509	0.185	0.571	0.524	0.189	0.606	0.570	0.313
	(0.023)	(0.024)	(0.045)	(0.025)	(0.025)	(0.046)	(0.023)	(0.025)	(0.052)
350–400	0.575	0.537	0.186	0.596	0.555	0.192	0.753	0.724	0.314
	(0.020)	(0.021)	(0.045)	(0.021)	(0.022)	(0.047)	(0.021)	(0.024)	(0.056)
400+	0.612	0.591	0.311	0.623	0.600	0.313	0.703	0.683	0.420
	(0.007)	(0.008)	(0.034)	(0.007)	(0.008)	(0.035)	(0.008)	(0.008)	(0.043)
Out-of-work	133.462	128.770	101.699	133.740	129.247	102.456	135.057	130.287	105.700
Income	(s1.672)	(s1.691)	(s3.112)	(1.701)	(1.700)	(3.219)	(1.781)	(1.757)	(3.077)
Hours bonus	–	–	–	44.652	40.775	16.021	47.734	48.347	32.962
				(1.801)	(2.260)	(3.225)	(1.507)	(1.700)	(5.230)
Hours point	–	–	–	19	19	19	33	33	36.5

Notes: Table presents optimal structure of marginal tax rates and out-of-work income under range of distributional taste parameters θ. All incomes are in pounds per week and are expressed in April 2002 prices. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table S6: The effect of random measurement error on the optimal hours bonus

Standard	$\theta = -0.4$			$\theta = -0.2$			$\theta = 0.0$		
Deviation	bonus	hours	welfare	bonus	hours	welfare	bonus	hours	welfare
0	44.06	33	2.24%	48.63	33	2.46%	51.70	40	2.44%
2	42.08	33	2.10%	46.48	33	2.30%	50.85	40	2.38%
4	38.28	33	1.82%	42.28	33	1.99%	43.53	40	1.82%
6	34.38	33	1.58%	37.82	33	1.71%	38.28	33	1.30%
8	28.26	33	1.22%	31.09	33	1.32%	31.49	33	1.02%
10	23.58	33	0.96%	25.73	33	1.03%	26.10	33	0.80%
12	21.55	26	0.77%	23.69	26	0.82%	22.88	33	0.68%
14	17.75	26	0.59%	18.33	33	0.63%	19.00	33	0.51%

Notes: Table shows how the optimal placement and size of hours contingent payments varies with random hours measurement error. Standard Deviation refers to the standard deviation of the additive independent normally distributed hours measurement error term. The columns "welfare" refer to the percentage increase in required expenditure to achieve the same level of social welfare compared to when no hours conditioning is performed. All incomes are in pounds per week and are expressed in April 2002 prices.

Table S7: Increases in expenditure to make no individual worse off

	Conditional on $(X, \epsilon, \varepsilon)$		Conditional on (X, ϵ_w)	
	No hours rule	Hours rule	No hours rule	Hours rule
$\theta = -0.4$	1.98%	2.56%	1.92%	2.03%
$\theta = -0.2$	2.52%	2.98%	2.50%	2.50%
$\theta = 0.0$	6.70%	6.34%	6.67%	5.95%

Notes: Table presents the increase in government expenditure required such that the value of maximised social welfare (under the additional requirement that no individual is made worse off conditional on $(X, \epsilon, \varepsilon)$ and (X, ϵ_w) respectively) is the same relative to when this constraint is not imposed.